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The Efficient Frontier and International Portfolio Diversification

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Abstract: This paper presents the main characteristics of the efficient frontier. In addition, it presents the essence of international portfolio diversification. It has been shown that this can be a tool for getting high returns on investments in stocks. Most important, however, is that the paper presents the possibility of building such portfolios consisting of international shares, which the efficiency frontier can be expressed by a mathematical function. To meet its function in practice, it is necessary to assume no short sales. Therefore, all combinations of portfolios have been prepared in such a case. This gives the reflection of a useful character. It should be noted that the used research methods include: research literature and empirical studies.

Keywords: portfolio management, stocks, expected rate of return, risk

Introduction

Portfolio Management from the perspective of capital investment should generate a number of benefits. The first is to achieve a high growth potential through properly selected investment assets. The second is to reduce the risk of investment while working out positive returns in all market conditions for a long-term investment horizon. The third includes liquidity, which represents a supply of cash which could be used to acquire assets with higher risk (in the case of the favourable conditions in capital markets). However, practice shows that in the case of equity investments, portfolio managers build portfolios without generating the benefits presented above. With all this in mind, it is worth noting that the management of the portfolio belongs to the field of knowledge, which is controversial. In addition to the above, they also derive from the fact that, in practice, in that case quite often capital investment's results are obtained with low values. Note that the annual rate of return of many risky investment fund's units, offered by operating in the Polish Society of Investment Funds, reach negative values (Fundi.pl).

All this means that new methods (models) are constantly sought, which would allow investors to achieve above-average returns on invested capital. One such method may be international diversification of the investment portfolio, which, in the era of globalization, it

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is increasingly used by portfolio managers. It should be made aware that the clearly rational character comes to the postulate that every investor incurring some risk wants to achieve the highest profit possible, or expected profit would be achieved at the lowest possible risk.

As it is known, the above combination of expectations is possible for the portfolios lying on the curve of efficient portfolios, known as the efficient frontier.

The main aim of this, is to answer the question whether the present time international portfolio's diversification is a practical tool that will allow to build joint-stock investment portfolios which offer relatively high rates of return (compared to the benchmark) with an acceptable risk. In addition to the above, the purpose of the paper is to present a combination of such portfolios that:

- are attractive to a potential investor (e.g. client of the wealth management industry),
- are on the efficient frontier, which accurately shows the relationship of the risk- return function described. This feature indicates that additional risk (expressed in percentages) must be paid so that the expected rate of return on a portfolio could increase by a predetermined value (expressed as a percentage).
- In this publication the following research methods were used:
- research literature,
- empirical studies of companies' share prices.

1. General characteristics of the efficient frontier

In the context of the management of investment portfolios there is an accepted assumption that the investor taking rational decisions chooses such efficient portfolios that, at a certain rate of return, offer the lowest risk, and for a certain level of risk – the highest rate of return (Markowitz 1952). The implication of the above are two main issues. Firstly, the risk must be measurable, so that investors could analyze investments characterized by different levels of risk. Secondly, investors characterized by different functions of utility, will be interested in different portfolios, but that will be offered in all possible combinations of the highest rates of return. It should be added that in the most famous model by Markowitz (Markowitz 1952), the risk of investment assets and portfolios is measured by the variance of their returns. These assumptions, while acceptance by the portfolio managers, became the basis of a range of effective investment funds in risky assets. The advantage of this is the ability to measure the risk of the portfolio, which, thanks to diversification, can be reduced.

If investors choose portfolios efficiently, they are only interested in the best portfolios of all possible combinations. A curve, on which these portfolios are located, is called the efficient frontier. „The efficient frontier therefore represents a set of portfolios, which gives the maximum rate of return for each level of risk or minimum risk for every level of return" (Reilly, Brown 2001: 375). For a better understanding of the presented concepts, the efficient frontier is presented in graphical form in Figure 1, assuming the absence of short selling and arbitrage.

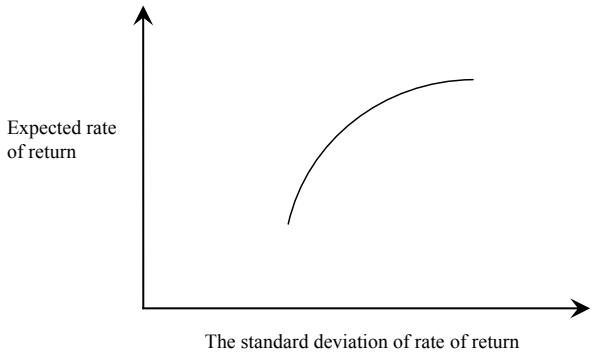

Figure 1. The shape of the efficient frontier

Source: own study based on (Albrecht Maurer 2008: 276).

Analyzing the efficient frontier, one should pay attention to its basic features, which include: continuity, convexity, and that it is growing (Yao and others 2013: 781–782). You also need to be aware that, how determined this curve is, it is affected by many factors, which include, among others: the number of investment assets, the yield of computer technology and the number of limiting conditions. However, in practice, the most common method used to draw the border is an effective method for quadratic programming (Michaud, Michaud 2008: 11).

It can be said that each individual investor chooses a portfolio that best meets his/her needs. This also applies to portfolios located on the efficient frontier. It has its justification in the fact that individual preferences play a significant role in investment decisions. Considering the management of portfolios, on the basis of the theory, the authors wish to point out, that the utility function characterizing the investor, determines the optimal composition of the portfolio chosen by him/her (Elton, Gruber 1998: 252). In practice, they prefer quadratic utility functions, and thus using the appropriate mathematical operations, you can find all the optimal portfolios lying on the efficient frontier (Kowgier 2006). However, the extent of the matter, while assessing the real essence of the presented approach, makes finding the optimal portfolios undetailed in the publication. In contrast, the next part is devoted to the international diversification of a portfolio.

2. The essence of international diversification of investment portfolio

The effects of diversification are well known and described in the portfolio theory developed by Markowitz (Markowitz 1952) and Tobin (Tobin 1958). This theory is not restricted to specific assets, but was mostly used to analyze diversification effects only domestically within one country. So, what makes the difference if the focus is international? Diversification

in the context of the portfolio theory means to reduce risk measured as a variance of expected asset returns at a given level of return by investing in additional international assets (Markowitz 1952: 89). As a next step, it has to be characterized what "international" is: International means investing in more than one nation. Following Adler, based on Solnik, a nation is a "zone of common purchasing power unit" or "subset of investors who use the same price index" (Adler, Dumas 1983).

In this context, it is of relevance, that the opportunity set of possible investment alternatives is enlarged extensively. First, internationally s more assets are available, e.g. shares not only in the American Stock market but as well in the European stock markets. Secondly, many more kinds of products are available. Typically the Portfolio Theory is discussed in the context of stock markets. But on the other hand assets are other products like bonds, country funds, American Depository Receipts, Exchange Trade Funds (like World Equity Benchmark Shares) or Hedge Funds too, which are not available in each country and could be integrated in the discussion as well. Sometimes these products lead even to a better result than stocks exclusively (Levy, Lerman 1988: 63).

Which are the major benefits of investing internationally? First, and most important, there is the advantage of risk reduction effects due to the number of securities, the size, internationality, industry of the firms (Rugman 1976: 75,79), the riskiness and correlation of their expected returns (Solnik, 1974). It could be shown that an international portfolio including all available assets is always dominating a domestic portfolio. This means that it is possible to find an international portfolio which is at the same return less risky than a local one, or earns more return at the same risk. Second, due to the fact that nations will not always move in the same direction (French, Poterba 1991), it is possible to reduce local risks of business cycles and other local weaknesses (Levy, Sarnat 1970: 668). Third, it is possible for international investors to take profit from arbitrage because of existing market imperfections like home biases (Gordon, Bovenberg 1996: 1057, 1073). Other advantages are the possibility of choosing a riskless alternative with a higher yield or just trading on markets with a higher liquidity than the local ones.

On the other hand diversifying internationally bears some risks: First, an investor has explicitly to take into account additional exchange risks which might lead to additional variances in expected returns of the home currency (Biger 1978: 65, 71). These risks could be hedged using adequate derivatives (Eun, Resnick 1994: 140, 160) and multicurrency diversification (Eun, Resnick 1988: 197, 214). Second, there exist political risks like discontinuities in the business environment, political changes, restrictions in profit repatriations and payment delays (Cosset, Suret 1995: 301, 303, 305, 315) which have an effect on returns and therefore on portfolio risks and should result in selection strategies. Third, transaction costs may be higher – especially in non-liquid markets- and other risks appear from tax aspects and limits on cross-border investments like shares restrictions (French, Poterba 1991: 224). So overall, each investor has to balance the chances and risks. While discussing all aspects of international diversification in all markets is much too complex, a reduction of

complexity is necessary. Therefore, following the discussions are concentrated on the effects of share returns, leaving other risks out of scope.

3. Presentation of the efficient frontier of portfolios consisting of Polish and German shares

For the presentation of an international portfolio's diversification as a practical tool that allows building investment portfolios that offer relatively high rates of return, the authors used shares listed on the Stock Exchange in Warsaw (Warsaw Stock Exchange) and the German Stock Exchange in Frankfurt am Main (Deutsche Börse). This justifies the choice of many arguments. First, the German economy was ranked 4th place in the world in terms of size and it is the largest in the European Union, and the Polish economy – the 24th largest in the world and is the highest among the new EU Member States (World Bank). Secondly, Poland and Germany belong to the same compound democratic states, which apply a uniform EU law that limits the legal risk of the investment. Third, the choice of investments in two markets reduces transaction costs (reconstruction of the portfolio). Fourth, the exchange rate risk was reduced because it includes only the relationship of one currency pair, i.e. EUR/PLN. Fifth, in this perspective, an international portfolio's diversification allows for the construction of portfolios consisting of companies listed on the stock exchange and mature characterized by low risk and with the companies listed on the stock exchange rising and offering a high rate of return, but characterized by a relatively high risk. Sixth, an international portfolio's diversification allows becoming independent of the stock market situation of a country. Seventh, the selected countries are characterized by moderate growth (Eurostat) and the high political stability (Political Instability Index).

Due to the nature of the study, one assumed the liquidity of the German and Polish stock markets and their high efficiency, which allowed making, selects stocks based on the criteria of risk and return. For this reason, the authors analyzed shares held by the indices covering the biggest in terms of market cap. In Germany, these include shares included in the index DAX30, Poland - belonging to the WIG 20 index. From the German index tested 30 shares, while the Polish - 15 shares, as against December 2007, not all of the 20 shares belong to the index.

The sole purpose of the efficient frontier, the average rate of return and the risk of Polish and German shares measured by the standard deviation, which is in accordance with the Markowitz rule (Markowitz 1952), calculated for the period 12.31.2007 – 02.28.2013. Theuthors analyzed the courses of shares' closure at each end of the month of that period, so the study was based on 62 courses of closures[1]. It should be noted that as the benchmark the DAX30 index was selected.

[1] The authors would like to add that the data was obtained from such news outlets as "Finanzen.net" and "Stooq.pl".

Finally, the objectives of building portfolios were as follows:

- refurbishment of the portfolio can be performed on a monthly basis (for more accurate calculations of the risk),
- the rate of return of the portfolio should evolve to a level higher than the benchmark at the level of the risk – the rate of return and the risk index of the German stock exchange - DAX (otherwise the international diversification, in practice, would not be profitable for investors)
- investment currency is the Euro, which will reduce the currency risk of investments and, therefore, prices of shares in the Polish study period were converted to values expressed in Euro (exchange rate taken from the closing date),
- transaction costs and portfolio management were not included, as well as the cost of purchasing the Polish Zloty currency.

Basic data and index values are in Table 1 (data on the Polish stock) and Table 2 (data on German shares).

Table 1

Risk and rates of return of Polish shares and Polish index in the period between 12.31.2007–2.28.2013 (monthly data)

No.	Name of the company (the order of the selected companies)	Ri (rate of return in %)	Si (risk in %)
1.	Asseco Poland SA	–0.49	8.72
2.	Bank Handlowy w Warszawie SA	0.66	12.09
3.	BRE Bank SA	0.58	15.62
4.	**Boryszew SA (3)**	**2.27**	**22.39**
5.	Globe Trade Centre SA	–1.97	12.77
6.	Jastrzębska Spółka Węglowa SA	no available	no available
7.	**Kernel Holding SA (4)**	**2.01**	**15.92**
8.	**KGHM Polska Miedź SA (2)**	**2.84**	**15.73**
9.	Grupa Lotos SA	1.02	16.32
10.	Lubelski Węgiel Bogdanka SA	no available	no available
11.	Bank Polska Kasa Opieki SA	0.26	13.11
12.	PGE Polska Grupa Energetyczna SA	no available	no available
13.	Polskie Górnictwo Naftowe i Gazownictwo SA	0.55	9.07
14.	Polski Koncern Naftowy ORLEN SA	0.93	10.72
15.	Powszechna Kasa Oszczędności Bank Polski SA	0.36	12.49
16.	Powszechny Zakład Ubezpieczeń SA	no available	no available
17.	**Synthos SA (1)**	**3.41**	**15.04**
18.	Tauron Polska Energia SA	no available	no available
19.	TELEKOMUNIKACJA POLSKA SA	–1.00	8.87
20.	TVN SA	–0.80	12.61
	WIG20	–0.29	7.22

Source: own study based on (Stooq).

Table 2

The risks and rates of return of German stocks and the German index in the period between 12.31.2007–2.28.2013 (monthly data)

No.	Name of the company (the order of the selected companies)	Ri (rate of return in %)	Si (risk in %)
1.	adidas AG	0.87	8.55
2.	Allianz	0.03	10.39
3.	BASF SE	1.01	9.43
4.	**Bayer (9)**	**0.55**	**6.83**
5.	**Beiersdorf (6)**	**0.55**	**5.94**
6.	BMW AG	1.34	10.15
7.	Commerzbank AG	–2.54	17.07
8.	Continental AG	1.59	16.75
9.	Daimler AG	0.09	11.97
10.	Deutsche Bank AG	–0.41	13.73
11.	Deutsche Börse AG	–1.20	9.72
12.	Deutsche Lufthansa AG	0.17	9.33
13.	Deutsche Post AG	0.08	10.37
14.	Deutche Telekom AG	–0.76	6.40
15.	E.ON SE	–1.74	8.71
16.	**Fresenius SE & Co. KGaA (St.) (10)**	**1.09**	**6.98**
17.	**Fresenius Medical Care AG & Co. KGaA (5)**	**0.70**	**5.02**
18.	HeidelbergCement AG	–0.06	13.47
19.	Henkel KGaA Vz.	1.17	7.06
20.	Infineon Technologies AG	2.55	26.02
21.	K+S AG	0.80	12.93
22.	**Linde AG (7)**	**0.91**	**6.68**
23.	LANXESS AG	1.96	13.06
24.	**Merck KGaA (8)**	**0.55**	**6.74**
25.	Münchener Rückversicherungs-Gesellschaft AG	0.21	5.58
26.	RWE AG	–1.64	7.76
27.	SAP AG	1.10	7.13
28.	Siemens AG	–0.10	8.83
29.	ThyssenKrupp AG	–0.40	12.90
30.	Volkswagen AG	1.79	13.24
	DAX	0.15	6.56

Source: own study based on (Finanzen.net).

In order to achieve high rates of return and achieve the goals, the authors selected four Polish shares of the expected monthly rate of return (expressed as a percentage and takes into account-exceeding conversion to the Euro) of not less than 2% without considering the risks. In contrast to the stability of the portfolio risk, the authors have chosen 6 of the German shares of the expected monthly rate of return of not less than 0.5% and the risk of not more than 7%. Table no. 1 and no. 2 for the names of companies currently in parentheses

order of selection for the portfolio (Polish shares have been classified according to the rate of return, the German – at risk).

On the basis of the calculations carried out by means of linear programming, the effective limit to afford all possible portfolios consisting of 10 selected shares, which is presented in Figure 2. It was assumed that the lack of short selling, which gives a useful character of the running considerations.

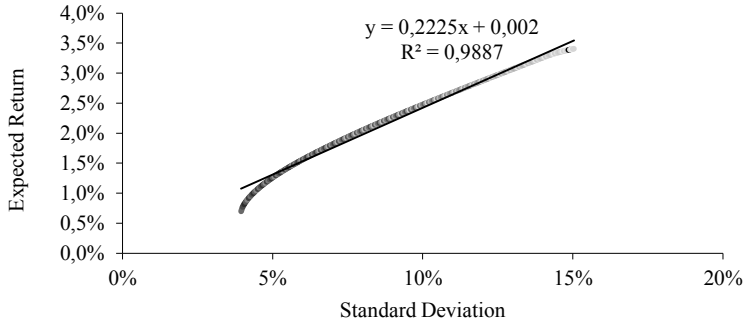

Figure 2. Efficient frontier of portfolios consisting of Polish and German shares

Source: own study.

The efficient frontier, which is the result of the study, adopted in such a form that it can be represented by the figure applied to the function (with a determination's coefficient of almost 99%). This function takes the form:

$$y = 0.2225x + 0.002y,$$

where:

y – is the expected rate of return on a portfolio composed of Polish and German shares?

x – is the level of risk that has to be incurred in connection with the investment? but $x \in \langle 3.95\%; 15.04\% \rangle$.

Thus, the function shows that with the increase in risk of 1% (on a monthly basis), in which the investor (client institutions offering wealth management services) agrees, the monthly rate of return of the portfolio should increase by 0.2225%. As you can see, the minimum risk they have to suffer through the optimal choice of selected shares is 3.95% (at the expected rate of return of 0.70%), and it is lower than the risk in the DAX index (6.56%) and lower than the risk in the WIG 20 index (7.22%). In contrast, the maximum expected rate of return derived from a combination of a certain action is up to 3.41 % per month (at the risk ratio of 15.04%). However, the most important information is such that the risk DAX30 index, or ratio of 6.56%, an international stock portfolio offers a rate of return of 1.71% on a monthly basis, where the expected rate of return DAX30 index is only 0.15%. Finally,

although this is not the most important issue, the combination of the portfolios included in the efficient frontier, authors counted volatility ratios. The lowest variation's coefficient among all possible combinations of portfolios amounted to 384.06% with a risk of 6.3% and a rate of return equal to 1.64%. As it can be seen, the lowest variation's coefficient relates to the combination in which the risk of the portfolio obtained nearly equal DAX risk. For this reason, the index can be considered as the benchmark.

Conclusions

This publication presents an international portfolio's diversification as an effective tool for obtaining high (compared to the benchmark) and expected rates of return. It was also shown that as a consequence of the appropriate selection of shares one is able to obtain such an efficient frontier that can be represented as a function. But the most important aspect is the fact that such a limit does not take into account the effective short selling, which gives it its practical value.

Of course, the level of complexity that accompanies the finding of such a bound indicates that this is not always possible, or may require a long period of time. But the most important aspect of an international portfolio's diversification is a positive effect on the effective management of the portfolio. It is expressed by achieving high rates of return and the selection of such investment assets that are correlated with each other to a small extent. As is known, the result of the selection of pairs of assets with low correlation is to reduce the overall risk of the portfolio. Given the above, we are of the view that an international portfolio's diversification can help in attracting customers in the wealth management industry, in the period in which they produce very low (or even negative) returns.

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GRANICA EFEKTYWNOŚCI A MIĘDZYNARODOWA DYWERSYFIKACJA PORTFELA AKCJI

Streszczenie: W niniejszej publikacji zaprezentowano najważniejsze cechy granicy efektywności. Ponadto, przedstawiono też istotę międzynarodowej dywersyfikacji portfela. Wykazano, iż ta może być narzędziem pozwalającym na uzyskiwanie wysokich stóp zwrotu z inwestycji w akcje. Najistotniejsze jest jednak to, iż w publikacji zaprezentowano możliwość budowy takich portfeli składających się z międzynarodowych akcji, których granica efektywności może zostać wyrażona za pomocą funkcji matematycznej. Aby spełniała ona swoją funkcję w praktyce, niezbędne jest założenie braku krótkiej sprzedaży. Dlatego też wszystkie kombinacje portfeli zostały sporządzone przy takim założeniu. To wszystko nadaje prowadzonym rozważaniom użyteczny charakter. Warto przy tym zaznaczyć, iż do wykorzystywanych metod badawczych zaliczają się: badania literaturowe oraz badania empiryczne.

Słowa kluczowe: zarządzanie portfelem, akcje, oczekiwane stopy zwrotu, ryzyko

Citation

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